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A Study of Chromatic Stereo-Effects

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A STUDY OF CHROMATIC STEREO-EFFECTS

by

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LIFE

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From 1944 to 1948 the writer was engaged in the practice of optometry in Linton, Indiana. From 1948 to 1950 he directed a series of clinical research studies in strabismus and amblyopia at the Chicago College of Optometry, Chicago, Illinois.

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The writer has published numerous technical articles in optometric publications and allied journals on the subjects of abnormal visual perception in strabismic children, three-dimensional seeing, retinal rivalry, flicker, television and the viewing of three-dimensional motion pictures.

Some of the articles he has written are:

Mirror Vision, Optometric Weekly, Nov. 11, 1948

Ambiguity in Ocular Dominance, (Co-writer J. Chrisakos), The Optician, London, England, Vol. 118; 3059, Nov. 18, 1949

Flicker Fusion Thresholds in Normal Persons, (Co-writer N. Fabricant, M.D.), Eye, Ear, Nose and Throat Monthly, Vol. 30, Mar. 1951

Relationship of Retinal Rivalry to Reading Achievement, Amer. J. Optom. & Arch. of Amer. Acad. Optom., Vol. 28; 11, Nov. 1951

Depth Perception; with Special Reference to Motion Pictures, Jour. Soc. Motion Pic. and Telev. Engrs. Vol. 60; 6, June 1953

The Vision of Children in Relation to Television, The Jewish Parent, Vol. 5; 5 and 7; April and June, 1954

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CHAPTER I

STATEMENT OF THE PROBLEM

Artists of the Renaissance identified "retiring and advancing" colors and some painters, acquainted with this phenomenon have used it as one of several cues in obtaining the illusion of depth in colored paintings.

According to Duke-Elder (4,1079), Kohlrausch (1871) noted that colors on a plain background appear to stand out in relief, noting that although the effect is seen with all colors, it is more marked the greater their separation in the spectrum. A specific observation has been made (1,5) that when red and blue are viewed in the same plane, the red appears considerably closer than the blue, with green and yellow objects occupying intermediate positions*. Changes in effect were noted with varying backgrounds.

Black and white were also noted to "recede and advance" when viewed against differently colored backgrounds. The hypothesis to be tested in this experiment is to determine whether such chromatic stereo-effects can be explained

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* This phenomenon is called chromatic stereoscopy.

on the basis of certain optical relations of the eye, based on its physical constitution. Elaboration of the hypothesis follows.

Hartridge (7,104) has prepared a table based on physical considerations to show the subjective effects produced by colors when viewed against a black and white background respectively (Table 1). Hartridge bases his explanation for the stereo-effect on the dispersion of rays at the surface of the cornea.

TABLE I

THE STEREOSCOPIC EFFECT PRODUCED BY COLORS*

Position	Color on a black background	Color on a white background
Nearest	Red	Blue-green
Nearer	Orange	Indigo
Slightly near	Yellow	Blue
Medium distance	Green	Purple
Slightly distant	Blue-green	Red
Farther	Blue	Yellow
Most distant	Violet	Yellow-green

* From Hartridge (7,104)

A similar observation was made as early as 1885 by Einthoven (4,774) who explained the effect in terms of chromatic differences in magnification of the objects which were viewed. He presumed that these differences were a consequence of the eccentric position of the pupil with respect to the fixation axis.

Both of these "explanations" are based on physical features of the laws of optics. Before these two theses can be elaborated further, or a contingent hypothesis formally stated, it is necessary to explain both the optics of the eye as well as the role of chromatic aberrations with respect to the locus of the differential stimuli mediating the stereo-effect.

Optics of the eye

In Figure 1, the optic axis, AP joins the anterior and the posterior pole of the eye, and on it are situated the centers of curvature of the refracting surfaces. The optic axis does not meet the fovea in most subjects. In the majority the fovea is temporal to the axial point P. The image of the object of regard, O, lies on the fovea, and the line joining the fovea with the nodal point projected from the eye strikes the object of regard. This line is known as the visual axis. The two axes have the nodal point in common, and the angle between these axes is called angle alpha.

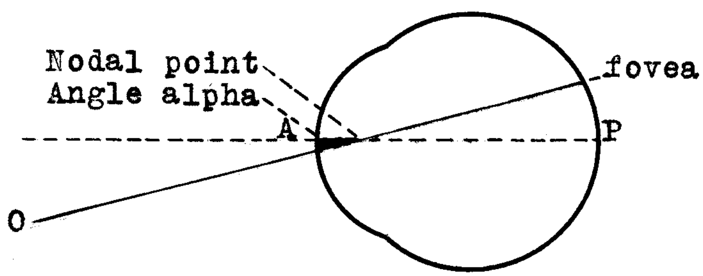


FIGURE 1

ANGLE ALPHA

Tscherning (15,79) determined the value of this angle by a special telescope with a double image prism, by lining up the images on the anterior surface of the cornea with the anterior and posterior images on the surface of the lens. Angle alpha is the angle between this line formed by the aligned images and the fixation axis. Tscherning found the value to vary between four and seven degrees in most subjects.

Angle alpha and chromatic aberrations

There are two ways of considering the effects of angle alpha on the course of rays of different wave length through the eye, advanced by Einthoven and Hartridge respectively. As light passes through the ocular media it suffers spherical (lenticular) aberration as well as prismatic dispersion. The former is responsible for the chromatic difference of magnification; the latter dispersion causes a lateral chromatic difference of focus.

Thomas Young in 1801 (2,110) (4,771) (12,201) was the first to note chromatic aberration in the lens system of the eye. Young estimated the amount correctly at 1.30 diopters, e.g., if the refractive power of the eye for red rays is 58.70 diopters (which corresponds to a posterior nodal distance of 1/58.7 or 17.04 mm.), then its refracting power for blue rays will be about 60.00 diopters (corresponding to a posterior nodal distance of 1/60 or 16.67 mm.). If the red rays are sharply focused on the retina, the blue rays will come to a focus in front of the retina by (17.04-16.67) 0.37 mm. See Figure 2.

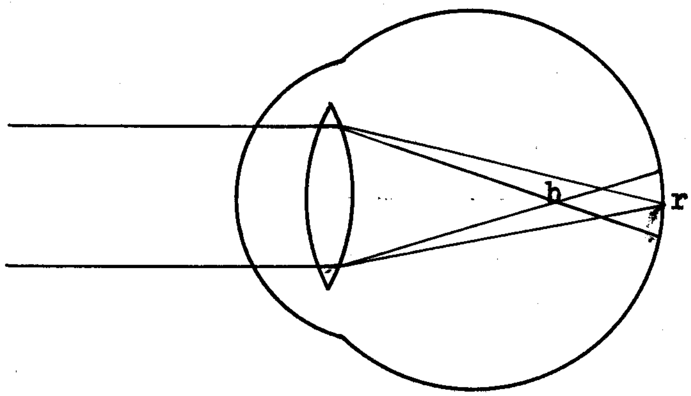


FIGURE 2

CHROMATIC ABERRATION

The correction for chromatic aberration has been attempted by Helmholtz (8), Javal (13) and others with little improvement in visual acuity. When a monochromatic light source is used, the correction for chromatic aberration is superfluous, but when the light is not monochromatic, e.g., a white light, a spectrum is formed on the retina, not a point of light.

Einthoven: chromatic difference in magnification

Even though their foci lie at different levels due to chromatic axial aberration as already explained, red and blue rays which enter the eye along the optic axis are concentrically represented on the retina only at the axial point.

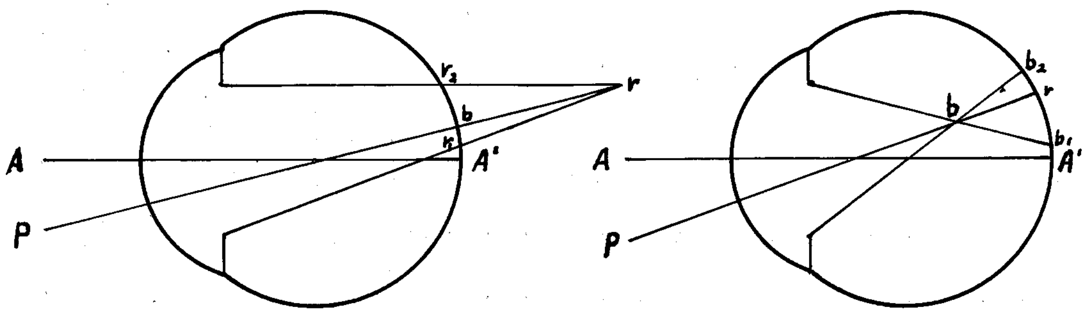
No spectra are formed at this point. This also is true for any combination of wave lengths.

Rays from white light which enter the eye along the visual axis follow an angular course with respect to the optic axis, and owing to differential prismatic dispersion, these rays form diffusion circles on the retina. Since the diffusion circles are not concentric, particularly in the case of the extreme red and blue rays, these rays form short spectra on the retina, instead of superimposed images.

The degree of eccentricity of the fovea from the axial point governs the length of the spectra, and the farther the image is formed from the axial point, the longer do the spectra become. The long axes of these spectra always go through the fixation point, and their short wave lengths are always closer to this point than their long wave lengths, so red rays occupy a temporal position, while blue rays stimulate points nasal to the fixation point.

According to Einthoven (4,774) these spectra are responsible for chromatic differences in magnification as shown in Figure 3. While blue rays are refracted more than red rays, the two sets of rays make different angles with the optic axis and thus stimulate disparate points.

In Figure 3(a), if the eye is focused for blue, the blue rays focus at b and the red rays at r. The diffusion circle on the retina $\underline{br_2}$ is greater than $\underline{br_1}$, and the red object will appear larger than the blue object. If the eye is focused for red (Figure 3b), $\underline{rb_1}$ will be greater than $\underline{rb_2}$ and the blue object will appear larger than the red one.



(a) (b)

FIGURE 3 (a) and (b)

CHROMATIC DIFFERENCE IN MAGNIFICATION

Duke Elder (4,774) also claims that this phenomenon is responsible for color stereoscopy, but points out that in ordinary vision when the eye is focused for yellow light, the error introduced is very small and yet, since the image of blue (due to greater refrangibility) is smaller than that for red, the error tends to correct the chromatic difference of focus.

Concordant with his argument, Einthoven (4,586) utilizes the eccentric position of the pupil with respect to the fixation axis to show that the resulting color stereoscopy is a function of the chromatic difference of focus. He calculates the linear separation on the retina for two rays at the ends of the spectrum.

A ray from the eccentric fovea with respect to the axial point, passing from the fovea to the posterior nodal point continues from the anterior nodal point eccentrically through the pupil. If the fovea is one mm. from the axial point, and the distance from the retina to the posterior nodal point is 16.8 mm., the distance between the nodal points is 0.3 mm., and the distance from the anterior nodal point to the plane of the iris is 2.7 mm., then the eccentricity of the fixation axis at the plane of the iris is 2.7/17.1 or 0.016 mm. If the focal points for the ends of the spectrum are 0.6 mm. apart as derived by Einthoven, their eccentricity would be, 0.016 x 0.6/19.7 equalling 0.0049 mm. This would be the linear separation of red and blue rays on the retina when both entered the eye on the visual axis.

The phrase "chromatic difference in magnification" may be better understood if it is supposed that the object in the visual field is circular and that the gaze is fixed on the center. The rays from every part of the circle will be deviated during transmission through the ocular media, the blue rays suffering more deviation than the red rays. In consequence, the blue circle will appear to be smaller than the red one, while the green circle will lie between them. Based on relative sizes, red objects which form the larger retinal image should be judged closer than blue objects, when both are located in the same spatial plane.

<u>Hartridge: differential dispersion</u>

Hartridge's claim for the stereo-effect is based on the differential refraction of the rays at the surface of the cornea (6,588). From a knowledge of angle alpha, the mean refractive index, and the dispersion of the ocular media, it is possible to calculate for any two spectral rays the angle which they will form with one another during their passage through the eye to come to a focus on the retina; e.g., if the rays are the C and F standard spectral lines, and the mean optical constants of the eye are equal to those of water at 20° C., as they are presumed to be, then the refractive indexes are shown in Table II.

TABLE II

REFRACTIVE INDEXES OF WATER AT 20° C.*

Spectral line	Index
r	1.3308
C	1.3311
D	1.3330
E	1.3352
F	1.3371
G	1.3406
v	1.3428

* From Hartridge (6,588)

If the distance between the fovea and the axial point is one mm., then the distance between the points at which rays corresponding to the <u>r</u> and <u>v</u> lines strike the retina is given by the formula,

$$\text{distance} = \frac{R_r - R_v}{R_d}$$

, where R_r, R_v and R_d are the refractive indexes for the red, violet and yellow lines respectively. By substitution (1.3428-1.3308)/ 1.3330 equals 0.0090 mm., which is about double that derived on Einthoven's basis.

The main premise developed from Hartridge's data postulates that the distance between red and violet (or blue) rays on the retina is a direct mathematical function of angle alpha. Thus in viewing two colored objects, one red and the other blue, the blue rays fall on nasal foveal cones, whereas the red rays fall on temporal foveal cones. Based on the cues of retinal disparity the reported percept is analogous to a stereoscopic percept. The measurement of angle alpha poses no special difficulty and any hypothesis to be proposed must include a consideration of this relation.

Gullstrand (6,589) who calculated the distance between the focusing points of the rays corresponding to the ends of the spectrum gave the value of 0.03 mm. at a distance of one mm. from the axial point. It is not known how this distance was obtained.

Table III contains a summary of the different values from the various authors. In each instance one mm. on the retina is equal to 297 cone units, the value being obtained from Goldman and Hagen's (6,590) measurement of the posterior nodal distance taken as 16.81 mm.

TABLE III

DISPERSION OF OCULAR MEDIA

Angle alpha is 1 mm. on the retina; 3.4 degrees, or 204 minutes of arc, with 297 cone units

Author	r to v	
	mm.	cone units
Gullstrand	.0300	8.94
Einthoven	.0049	1.46
Hartridge	.0090	2.68

<u>Chromatic difference in magnification</u>

It has been shown that the actual value of chromatic difference of focus at any point on the retina depends on the distance of that point from the axial point and the magnitude of the dispersion suffered by the light rays as they traverse the ocular media.

In practice for an object of finite size, the value is a percentage of its linear dimensions. The value of the percentage depends on the wave length of the rays forming the images of objects. The percentages at different wave lengths calculated by the writer on the basis of the data from Gullstrand, Einthoven and Hartridge are shown in Table IV.

TABLE IV

CHROMATIC DIFFERENCE OF MAGNIFICATION

Author	6500 - 4000 A°
Gullstrand	3.00 %
Einthoven	.49 %
Hartridge	.90 %

A discrepancy appears at this point. According to Einthoven, the chromatic difference of magnification for rays corresponding to the ends of the spectrum probably has the value of 0.5 per cent. If Goldman and Hagen's value for the posterior nodal distance of 16.81 mm. is used, then angle alpha which varies from four to seven degrees (Tscherning) is found to correspond to a distance of 1.5 and 2.1 mm. between the foveal center and the axial point. This is based on the assumption that Einthoven's value for the chromatic difference of magnification is correct. There is uniform agreement that the distance between fovea and axial point is only one mm., which indicates that Tscherning's values are too high. This is a point which should be verified during the course of this experiment.

Current status of problem and hypothesis

In summary, the current status shows unanimity of observation that color acts as a spatial cue. Three tentative "explanations" can be advanced as to the basic mechanism involved.

The first of these is that varying amounts of accommodation required for critical focus of a colored object may serve as the cue. Einthoven's thesis is based on the chromatic difference in magnification, in that the larger retinal image (red) is a cue to nearer objects as compared to a smaller retinal image (blue). Hartridge postulates that the chromatic stereo-effect is due to a differential refracting of the light rays at the surface of the cornea resulting in a stimulation of disparate retinal elements which elicit the response on the basis of stereopsis.

It is well known that color perception as well as space perception are highly subjective phenomena. No thesis has been advanced which postulates that stereo-effects may vary from one person to another independent of optical status, or that the effects may fluctuate in the same person from one time to another, not only in degree but also in the quality of the stimulus. No attempt has been made to show whether there is any subjective interaction with a particular color on a psychological basis.

The accommodative theory is a physiological approach, and accommodation may readily be controlled by selecting an appropriate distance, by a refractive correction and by the use of selected stimuli.

The views of chromatic magnification and chromatic stereoscopy are based on the angle alpha relation. This can be measured accurately and should be correlated with experimental test results. No one seems to have attempted this

If these "tentative explanations" are demonstrated to have no causal relation with the test results, and no other alternatives are available, the stereo-effect, being based on highly subjective perceptions may be ascribed to psychological factors. An adequate analysis of variance would be required to indicate just where the source of variation exists, and how much of it is due to interaction.

If dispersion of chromatic stimuli (colors) is present in one or both eyes of the observer, the phenomenon of color stereoscopy should become manifest and it should correlate with angle alpha. The hypothesis to be tested in the present experiment is to determine whether chromatic stereo-effects are a function of angle alpha and follow the spectral order of arrangement as predicted from optical calculations of chromatic aberrations in the eye.

Some basic considerations

In selecting subjects for this study, attention should be centered on two requisites pertinent to stereo-effects. The two visual functions involved are good stereoscopic vision and high acuity for fine detail. Normal stereoscopic vision is increasingly more effective the nearer the object approaches the observer and it fails at great distances owing to the reduced range-base provided by the interocular distance. Not so in chromatic stereoscopy which therefore becomes relatively more valuable the farther the objects.

However there is a distinction. Normal stereoscopic vision operates independently of color and therefore black and white objects are not involved in chromatic stereoscopy. The relationship of black and white objects is mediated by other depth cues, one of these being that of relative brightness. Duke-Elder claims that the value of chromatic stereoscopy is such that it operates for objects of all sizes at all distances.

Insofar as acuity is concerned, there are effects due to displacement of the aberration discs of different wave lengths relative to one another. At the optical axis the discs are superimposed and their centers coincide. Away from the optical axis the centers are displaced sideways, the short wave length discs are closer to the axial point than the long wave lengths. As the distance increases a stage is reached where the relative displacement attains a value where the discs no longer overlap. The retinal image now has the shape of a figure-eight, one loop formed by the short wave lengths and the other by the long wave lengths. The medium wave lengths (yellow color) are at the junction of the two loops and fall on the fovea (3,427).

The third fact which emerges from this consideration is that if stereo-effects are to take place at all, the long wave lengths must fall on corresponding retinal points while the short wave lengths must not do so, and vice versa. This relationship is presumed to be a normal consequence of the angle alpha value.

CHAPTER II

REVIEW OF RELATED LITERATURE

There appears to be a scarcity of research in this field. Neither Einthoven or Hartridge present experimental findings which bear directly on this problem. A search of the literature prior to 1918 failed to yield any experimental information on chromatic stereo-effects. In 1918 Luckiesh (10) reported the results of an experiment in which he used monochromatic red and blue letters formed by masking filters of high purity.

Ten observers made ten estimations by viewing the letters simultaneously against a dark background at each of six distances ranging from two and one-half meters to seven and one-quarter meters. Each observer adjusted the red stimulus to the subjective plane of the blue stimulus.

In most instances the red letter was moved farther away, and this trend generally increased with the distance of the observer's eyes. Luckiesh himself saw the red closer at the two and one-half meter distance by 0.39 cm. (fourteen per cent), and the distance increased to 1.27 cm. at the seven and one-quarter meter distance, while the percentage decreased to two and seven-tenths per cent. The phenomenon was very definite in most of his other observers. Ordinarily chromatic aberration alone could account for this effect, but consistent negative values for two of his nine cases and other considerations complicate this explanation.

Luckiesh concludes that the colors actually "retire and advance" subjectively, and that red is the advancing and blue the retiring color. If this explanation is correct the distribution follows the spectral arrangement and could be explained in terms of prismatic effects on the retina. (One must note that Luckiesh used a tungsten lamp source for transillumination and he apparently failed to equalize the luminosities of the two stimuli). Luckiesh admits that differences in relative brightness and the shape of the letters seemed to influence the magnitude of the results, but the effect was never reversed, not even in his two contradictory cases. The effect could not be observed with certainty when one eye was used which suggests that a binocular stereoscopic judgment was involved.

Taylor and Sumner (14) have experimented and claimed that they controlled the brightness of their colored targets. They report that when the apparent (subjective) distances of different colors are held constant (subject sees the colors in the same spatial plane), the actual distances of these colors differ from one another (on the objective side) and that the amount of the disparity may be designated as the error of distance for that color.

This was an inadequate study for they used Hering papers which were illuminated with a forty-watt fluorescent lamp mounted in a Howard-Dolman apparatus. Since the various papers had different reflectance values varying from seven per cent to seventy-six per cent, brightness was an uncontrolled variable in color discrimination.

Taylor and Sumner did not have to "prove" that at a constant distance the light colors (white, yellow and green) appeared closer than the dark colors (red, blue and violet). This effect is simply a function of brightness values and could have been predicted. The writers did stress the need of considering the factor of a figure-ground relativism of Gestalt psychology in explaining some of their effects, but failed to realize the stereo-effects due to hue also.

Karwoski and Lloyd (9) conducted a study on the role of chromatic aberration in depth perception using Wratten filters with high cut-off points. Although significant differences in responses were scarce in the data, they claimed that the trend of the ratios indicates that the farther apart the spectral colors, the more reliable the differences. To make this claim they had to discard three of their ten subjects who showed reversals; in addition, one of their subjects had subnormal vision, three were colof-blind, and others had visual errors of hypermetropia and astigmatism which were not even measured.

In all these studies uncontrolled variables are present; these are brightness values, target differences, type of illumination with respect to color stimulus, subnormal vision in some subjects, color blindness in others, and varying refractive errors which were not measured.

These variables could be important in causing certain results in specific cases, and their varying magnitudes could easily explain the negative and apparently contradictory values in others. For these reasons it becomes imperative that appropriate controls be instituted with respect to these uncontrolled variables.

CHAPTER III

EXPERIMENTATION

Preliminary experiment

In order to determine the design of the controlled experiment is was deemed desirable to conduct an exploratory study for orientation on the basis of subjective responses to chromatic stimuli viewed on different backgrounds. The purpose was threefold; (1) to determine whether luminosity (color brightness) would interfere with a response based on chroma alone, (2) to determine what trend was present in a group as a whole, and (3) to determine whether individual consistency was present.

Two pieces of poster-board with a semi-gloss finish, one white and the other black, each measuring twenty-two by twenty-eight inches were used as the background against which a series of colored discs were viewed simultaneously. Six colored discs were prepared from Stratochrome paper (a dull-coat paper with relatively little gloss). Two sets of the discs were prepared. Each disc had a diameter of three and three-quarters of an inch, and the set of six was pasted on the poster-boards, equidistant with a center-to-center separation of eight inches. There was a four-inch separation between the edges. The colors of the discs were red, orange, yellow, green, blue and purple.

Seventy-seven subjects were asked for a phenomenal description of the spatial order of the colored discs viewed first against the black background, then against the white background. The discs were viewed under daylight illumination at a distance of twenty feet.

The results of the responses are tabulated in Appendix I. Only the most proximal and the most distal colors are listed respectively. On the black background forty-two of the seventy-seven subjects selected the yellow as the nearest disc, and forty-four selected the blue disc as the farthest one. There were eleven categories of response with the bulk of the group falling to the "yellow-blue" group. Ten of the seventy-seven subjects experienced such fluctuations in the spatial order of colors that they could not identify the nearest or farthest color with any degree of certainty.

On the white background thirty-one of the seventy-seven subjects selected the blue as the closest and fifty reported the yellow as the farthest back. Those falling into the "blue-yellow" group were twenty-three in number. Two subjects reported no effect, and seven others showed such alternations in spatial order that they could not give a reliable report.

Study of the data showed no common denominator to be operating consistently as regards the spatial order. A strong trend was noted in that there was a tendency to identify the displacement on the basis of brightness contrast rather than chromatic effect in both instances. On both backgrounds the nearest target reported by the greatest number of persons showed the greatest contrast whereas the farthest target showed the least contrast. While these results are not concordant with the theories advanced on pages 13 and 14, it is still possible that the theory was predicated on the assumption that the various colors had equal luminosity values.

In this experiment had the stereo-effect been predicated only on the basis of brightness cues or of chromatic cues, no more than two types of responses would have been expected, namely (on the black background) yellow-green-red-blue for brightness, and red-yellow, green and blue for hue. Since there were so many varied responses, this indicates that the perception of depth as influenced by colors and their luminosities is not simply an influence of chroma or brightness, but that these and more subtle factors play a part. Some of these may be psychological factors associated with individual differences of many sorts which have not yet been fully explored.

These results suggest that brightness contrast is an interacting factor in identifying stereo-effects; that there is marked variability between individuals, with some inconsistency in the same individual as shown by certain fluctuating situations of alternation, and finally that in any spatial experiments dealing with chroma, it is important that luminosities be controlled.

Apparatus for the controlled experiment

Bearing in mind that we are testing the hypothesis that chromatic stereo-effects are a function of angle alpha it is imperative that we institute appropriate controls over those variable factors which might invalidate the data and defeat the purpose of the experiment. In this experiment the differentiating independent variable is the series of primary hues used as the stimulus. The dependent variable is the spatial alignment of each pair of stimuli by the subject, which can be translated into numerical values by the method of average error.

Other independent variables (causal factors) which must be controlled or known through measurement are the visual status of the subjects a totally darkened room, a constant test distance of twenty feet, uniformity of instructions, the degree of naivete of the subjects regarding the purpose of the experiment, and a presumed healthy motivation because the subjects were all mature students who had volunteered for the experiment.

The apparatus for this experiment was designed precisely in order to appraise chromatic responses of subjects in response to stimuli of selected spectral bands having equal luminosity or brightness values.

Three wooden boxes were constructed, each two and three-quarters inches wide, six inches high and eight inches long. The inside was painted with a flat white (non-glossy) paint, and the outside was painted will a black non-glossy paint.

At one end of each box a window one and three-quarters inches square was cut out and a white diffusing glass was mounted in this aperture on the inner side. Selected Wratten filters , each forming an exposed colored square, exactly one and three-quarter inches, were placed in front of each diffusing glass. An air-space of one-quarter inch between the diffusing glass and the filter prevented the filter from becoming heated.

The back end of each box was equipped with a variable rheostat (resistance 1500 ohms) to control the light intensity of a light bulb ranging from fifteen to twenty-five watts. The light socket and bulb were four inches behind the plane of the filter. The light bulbs were manufactured by Westinghouse, the red and green filters were illuminated by fifteen watt bulbs, while the blue filter (the control) was illuminated by the twenty-five watt bulb.

The boxes were arranged on separate tracks, side by side, with a center to center separation of six inches. Each box could be moved to and fro independently by the observer by means of strings and pulleys. It was possible to change the lateral position of any of the three boxes with respect to the others with ease.

Three subjective color stimuli were produced by using filters with high cut-off points. The three primary hures were produced by combining filters as shown in Table V.

The red was produced by using Wratten filters twenty-nine and thirty-five; the green by using filters sixty-one and seventy-seven, and the blue by using filters forty-five and forty-seven. The transmission ranges of the various filter combinations with their selective filtering results used for stimulation were calculated by the writer and appear in the Table.

The range and the peak of the curves were verified by a grating spectrograph; the stimuli proved to be monochromatic with no overlap, with the peak of the transmission curve as indicated. Since the filters varied in terms of light transmission, the total light transmission was calculated and appears in the last column of the Table based on an arbitrary unit of 1.00 .

The last column shows that the luminosities are not equal, green being the brightest and blue the darkest. In order to obtain maximum light transmission of the blue it was necessary to use a twenty-five watt bulb and this was used as the standard to which the other brightnesses were adjusted. The three lights were equalized eventually by varying the rheostat adjustments of the red and the green stimuli.

TABLE V

TRANSMISSION VALUES OF FILTERS

Wratten color filters		Wave-length transmission (mμ)*	Range	Peak of curve	Per cent of light transmission
Red	29	610-700	650-700	690-700	.56
	35	(650-700 (400-470			
Green	61	490-600	520-600	550-560	5.29
	77	520-700			
Blue	45	430-540	430-510	450-470	.16
	47	370-510			

* From Wratten Light Filters, Seventeenth Edition, Revised by Eastman Kodak Co., Rochester, N.Y., 1944, pp. 75-83

Equalization of luminosities of the stimuli

It is apparent from the work of Luckiesh (10) and that of Taylor and Sumner (14) that brightness is a complicating factor in experiments dealing with colors. This was substantiated in the results of the preliminary experiment, where no attempt was made to control simultaneous brightness effects. Middleton (11) in working out an ICI formula for apparent color in terms of chromaticity and brightness found that the spatial visual range of colored objects does not differ greatly from achromatic objects of equivalent brightness.

As regards luminosity (brightness of colors), it is known that the intensity of the light sensations produced by different lights is not only a function of the light stimulus itself, but also varies with the wave length. However, the luminosity of a color sensation is susceptible of quantitative estimation on a scale of luminosities, for according to Abney's Law (4, 873) the luminosity of the combined spectrum is equal to the sum of the luminosities of its parts.

There are four methods of equalizing light differences. Heterochromatic photometry was employed in equalizing the luminosities of the spectral filters. The method of flicker was selected. This method is based on the fact that when rotating sectors are travelling at a certain speed a rapid succession of visual impressions are received producing a sensation of flicker, but if the speed is increased these sensations become fused into one continuous sensation.

The point at which the intermittent stimuli give rise to a continuous sensation (called the critical fusion frequency) is a simple function of the luminosity and is independent of hue, so that if flicker ceases at the same speed of rotation for two lights, then their luminosities are equal (16, 192 and 201). The application of this measurement was first suggested by Talbot

(1835) and subsequently applied by Ferry (1892), Rood (1893) and others, according to Duke-Elder (4,875).

In this experiment the two lights were separately alternated with black by means of a rotating sector; they were regarded as of equal luminosity when the flicker disappeared at the same speed of alternation (16,195).

A sector subtending an angle of twenty-five degrees was cut out of a metal disc twelve inches in diameter. The disc was painted a flat black and mounted on a color wheel. The light of lowest brightness (blue) was placed alongside one of the other two colors, both lights being behind the color wheel mixer. The centers of the light patches were six and one-half inches apart and the observer could see the two stimuli alternately as the wheel revolved. Flicker was noted. The revolutions were slowly increased until the observer, sitting twenty feet away reported an absence of flicker in the blue light. The variable rheostat was used to reduce the light intensity of the brighter light until that flicker just disappeared. At this point the luminosities of the two lights were presumed to be equal. On decreasing the revolutions of the disc flicker appeared simultaneously on both sides.

This calibration on the light control rheostat was performed with each set of lights on alternate sides with at least six observers. The end-points of flicker disappearance and reappearance were used for the calibration. The greatest variability of the observers was only two degrees of the rheostat setting, which represented a variability of ten ohms. Compared to the total resistance introduced (1163 ohms) to neutralize the luminosities this represents a maximum variation of eighty-six hundredths per cent of the total resistance. Although the number of revolutions for one observer differed from that of another, due to threshold differences, all subjects were in close agreement on the amount of light reduction necessary to abolish flicker in a given pair of lights.

Phenomenal responses to the test situation

Before undertaking quantitative measurements on a series of subjects, sixty-one naive observers ranging in age from ten to sixty years of age were asked for a phenomenal report of the spatial order of these lights. The three test stimuli of monochromatic squares were set up in a darkened room in the same spatial plane. The test distance was twenty feet.

All subjects were known to possess binocular vision with varying degrees of acuity. Their responses were recorded. After a period of time, varying from seven days to three weeks the same observers were asked to repeat their observation and the responses were recorded. Only fifty-two of the original sixty-one returned for the second observation, so nine of the original observations were discarded in the final tabulation.

Since at no time was green mentioned as the closest color, three types of responses were obtained in test and retest; either the red was closer, or the blue was closer, or there was an alternation between the red and the blue.

Table VII shows the percentages of the responses on the initial and final observations. In both cases the percentages of those identifying the blue as closest is about sixty per cent. However of those who saw the red closer (or the blue), on the first observation not all identified consistently on the final observation. Twenty-nine per cent identified the red in both tests; fifty-two per cent reported the blue in both tests and four per cent of the subjects showed continual fluctuations in both tests. Eight of the fifty-two subjects (fifteen per cent) showed a change from the initial response, moving from one of the three classes of response to one of the other two.

TABLE VI

PERCENTAGES OF PHENOMENAL RESPONSES

Phenomenal report	Initial observation	Final observation	Concordance
Red closer	32.7	36.5	28.8
Blue closer	61.5	59.6	51.9
Alternation	5.8	3.9	3.9
Total	100.0%	100.0%	84.6%

The controlled experiment

This experiment, designed to yield quantitative results, was administered to nine persons who had not been used previously for any observations. The subjects were all mature male students at a professional school* who volunteered as subjects and were selected on the basis that they had free time to participate in the experiment.

Each subject was required to pass all the plates in the Isha Hara color discrimination test and the Keystone (DB-6 stereogram) stereopsis test using the telebinocular. Each subject had a visual acuity of 20/20 in each eye or better. Three of the subjects wore no habitual correction, and the remaining six wore their prescribed lenses for all parts of the experiment. The refractive errors were noted and the pupillary distance (interocular separation) was measured and recorded.

Angle alpha was measured on each subject by means of the Brombach perimeter. The eye not under measurement was occluded with an eyepatch. The subject looked at a reflection of his own eye which he saw in a small mirror attached to the center of the perimetric arc.

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A tiny light source was moved along the arc with the experimenter's eye just above it. The experimenter peeped through a small telescope on the arc of the perimeter. By these means two catoptric images were seen, one on the anterior surface of the cornea and the other on the anterior surface of the lens. As the target light was moved laterally it reached a position where the two images appeared in superimposition. (Actually they were one behind the other). At this point angle alpha value was read off the arc of the perimeter. The angle was measured on the other eye, for purposes of comparison and averaging, but in all cases save one the angle was found to be the same. The one showed a questionable difference which varied about one degree, and this difference was ignored. The data on the visual status of the subjects is summarized in Table VII.

Following this, each subject was seated in a totally darkened room twenty feet from the plane of the light stimuli. The three lights were switched on simultaneously and the subject was instructed:

> You now see three squares of colored lights. Tell me their relation in terms of the distance from you, i.e., whether they all appear in the same plane, or whether any appear spatially displaced along the *z** axis.

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* Since all participants had taken courses in geotrical optics, they were familiar with the meaning of "*z* axis".

TABLE VII

VISUAL STATUS OF THE SUBJECTS

Subject	Angle alpha (degrees)	Pupillary distance (mm.)	Refractive status corrected 20/20
KO	4.0	65	Plane
BE	2.0	64	-3.00-3.00 -2.75-3.50
LU	3.5	63	0.50-0.50 0.50-0.50
BR	3.5	65	0.75-0.50 0.75-0.50
PE	4.5	66	-0.25-0.25 -0.50
TR	3.5	66	-4.00 -4.50
DA	0.0	64	Plane
AR	3.0	63	Plane
TA	2.0	63	-0.25 -0.25

Each subject promptly reported a staggered spatial arrangement which was immediately recorded. Two of the subjects reported some alternation in the spatial arrangement.

One of the three lights was extinguished in random order. The order was determined by tossing a die into a small box and if:

a. the one-spot turned up, the blue light was extinguished and the green was pushed forward,
b. the two-spot turned up, the blue light was extinguished and the red was pushed forward,
c. the three-spot turned up, the green light was extinguished and the blue light was pushed forward,
d. the four-spot turned up, the green light was extinguished and the red light was pushed forward,
e. the five-spot turned up, the red light was extinguished, and the blue was pushed forward,
f. the six-spot turned up, the red light was extinguished, and the green was pushed forward.

The two lights always occupied adjacent tracks. Any given light was pushed forward a distance of two meters (toward the subject) and he was asked to adjust the displaced light to spatial equality with the other light. The instructions were of a stereotyped nature:

> Select the proper strings corresponding to the color patch you want to move and manipulate them till you align this light with the stationary one, which we shall call the standard, for the time being.

After this had been accomplished the same light was moved backward (away from the subject) and he again adjusted this light to apparent equality. After the settings had been recorded the procedure was repeated using the previous "standard" as the movable target. After these four readings were obtained, the entire procedure was repeated with the lateral position of the two lights reversed. In all, eight readings were taken for each pair of lights, making twenty-four responses for each subject. After the initial eight responses were recorded, the die was tossed again to determine which of the two remaining lights was to be extinguished, and the same order was followed. When necessary the die was tossed two or even three times at this stage.

The psychophysical testing method employed in this experiment is known as the method of average error. The readings for each pair of stimuli were averaged to yield the "constant error" for each subject. This constant error is assumed to be an index to the subjective spatial difference of the two colors for that subject when the colors are reported to lie in the same plane phenomenally. Relative comparative results of the three sets of stimuli-combinations are discussed in Chapter IV.

CHAPTER IV

ANALYSIS OF RESULTS

The results of the preliminary experiment, based on seventy-seven subjects indicate that brightness was an independent variable which had to be controlled in studies of chromatic stereo-effects. Marked variability between individuals, as well as some inconsistencies in the same individual indicated the possible presence of factors other than brightness or chroma.

The results of the phenomenal responses to the test situation based on fifty-two subjects (Table VI) indicate that even when brightness is controlled and equalized for experimental purposes subjective differences still are present with respect to color. Fifteen per cent of the subjects showed a change from their initial observation over a period of time which varied from seven days to three weeks.

It may be recalled that none of the fifty-two subjects reported green as the nearest color. Is this distribution of scores a deviation from chance expectancy ? To answer this question, the chi square test can be used, with a five per cent allowance for a response of alternation, uncertainty or of no effect. Table VIII shows the analysis where X^2 is found to be over thirty-one. With three degrees of freedom a X^2 of 11.341 is significant at the 1% level of confidence and indicates (provisionally) that red and/or blue are significantly selected as the most proximal color over green.

TABLE VIII

CHI SQUARE ANALYSIS OF DATA BASED ON FOUR POSSIBLE RESPONSES

Color	fo	fe	fo-fe	$(fo-fe)^2$	$\frac{(fo-fe)^2}{fe}$
Red	17	16.5	.5	.25	.0152
Blue	32	16.5	15.5	240.25	14.5609
Green	0	16.5	-16.5	272.25	16.5000
?	3	2.6	.4	.16	.0616
? indicates alternation					X^2 = 31.1377

It is possible to apply the chi square test for a deviation from chance expectancy to the scores tabulated in the three classes of responses which were given. This analysis is shown in Table IX.

TABLE IX

CHI SQUARE ANALYSIS OF DISTRIBUTION
ON BASIS OF THREE RESPONSES OBTAINED

Color	fo	fe	fo-fe	$(fo-fe)^2$	$\frac{(fo-fe)^2}{fe}$
Red	17	24.7	-7.7	59.29	2.4004
Blue	32	24.7	7.3	53.29	2.1575
?	3	2.6	.4	.16	.0616
? indicates alternation				$X^2 = 4.6195$	

With two degrees of freedom X^2 should be at least 5.991 to be significant at the 5% level of confidence. Our obtained chi square is only 4.6195, which could very well be a distribution purely by chance. We cannot say that either red or blue is identified as closer on a statistical basis.

Regardless of the outcome of these two analyses, one should be very cautious in interpreting this data because these small degrees of freedom cause the sampling distribution of chi square to be greatly skewed.

The results of the controlled experiment are found in Table X. The subject's initials (in the first column) are followed by the subjective description of the spatial order, with the nearest test target listed first and the farthest listed last. The derived average settings for the three pairs of stimuli are in tabular form, and the last column shows consistency or inconsistency of the spatial measurements. All figures are given in millimeters and the same meaning attaches to all parts of the table.

TABLE X

SUMMARY OF SUBJECTIVE RESPONSES

Subject	Spatial order	Relationship of settings "Constant error" in mms.			Consistency
KO	RGB	R 241 B*	R 105 G	B 104 G	?
BE	RGB	R 177 B	R 135 G	G 4 B	**
LU	RGB	R 78 B	R 62 G	G 11 B	**
BR	BRG	B 212 G	B 210 R	R 2 G	**
PE	BRG	B 89 R	B 58 G	G 35 R	**
TR	BGR	B 94 R	B 73 G	G 41 R	**
DA	BGR	B 189 R	B 135 G	G 56 R	**
AR	BRG GBR	G 12 R	R 14 B	B 4 G	?
TA	BGR R-B -G B-R -G	R 33 B	R 4 G	B 10 G	?

* When red and blue are phenomenally in the same plane, the blue is objectively 241 mms. closer to the subject, which means that when they are in the same objective plane, he sees the red 241 mmss closer than the blue.

** Indicates consistency in measurement and spatial order.

? Indicates a contradiction between subjective description or in measurements.

As an example, subject KO identified the red as closest, the green intermediate and the blue as the farthest away on the initial exposure of the stimuli. In his red-blue matches final analysis of KO's settings shows that he sees the red 241 mm. in front of the blue; in the red-green matches he sees the red 105 mm. in front of the green, and in the blue-green matches the blue is seen 104 mm. in front of the green.

Since the first two matches tell us that blue should be seen farthest away, and since the subject did in fact report the blue as farthest away initially, it is a contradiction for him to see the blue 104 mm. in front of the green. The question mark indicates the contradiction.

Discussion of results

Three subjects, KO, BE and LU identified the red as closest with the blue farthest away. Measurements of the relation showed that two of the subjects, BE and LU set them in this order although they varied in amounts and percentages. The third observer, KO, gave a contradictory set of findings as already described.

Two of the subjects, BR and PE, identified the blue as closest with green the farthest. Although they were consistent in their settings they varied markedly in their comparative distances and percentages. Two other subjects, TR and DA, also saw the blue as closest, but saw the red farthest away. They too, were consistent in their settings, but differed markedly in their distance and settings in percentages.

Subject AR alternated in his report, once seeing the blue closer with the green farthest, then the green closest with the red farthest. This instability kept operating during all his settings. Results of his settings shows an impossible condition of green being closer than red, red closer than blue, and blue closer than green.

Subject TA had a great deal of difficulty throughout. His identification of the nearest color alternated between blue and red. At first he saw them in the order blue-green-red, but after a time red was the closest with blue and green the same distance behind; later when he saw the blue as closest, the red and green were the same distance behind and would even alternate from time to time. Analysis of his settings also shows the contradiction of seeing the red a little closer than the green, the red a great deal closer than blue, but seeing blue closer than green. There were instances when TA saw all three in the same plane for minutes at a time. The inconsistent response of his settings agrees with his initial responses.

We now have the experimental data in such form that we can apply it to the test of our hypothesis, is the spatial order of chromatic stereo-effects a function of angle alpha ? If not, is it any way related to pupillary distance or the refractive status ? A partial answer can be obtained to this question by a correlation analysis of the data of Table VII with that of Table X. Due to the small numbers involved, and the ease of ranking all the findings under consideration, the Spearman rho was computed. The results appear in Table XI.

TABLE XI

SPEARMAN CORRELATIONS BETWEEN STEREO-EFFECTS AND OPTICAL MEASUREMENTS

Combination	Angle alpha	Pupillary distance	Refractive status
Red-green	.100	- .185	- .067
Red-blue	.121	- .296	- .217
Blue-green	.136	.587	.300

When N = 9, 5% level is .666
1% level is .798

From the above data it is apparent that none of the correlations are significant. The angle alpha relations run from a low of .100 to a high (?) of .136, which means that the hypothesis advanced, that chromatic stereo-effects are a function of angle alpha has been disproved. It is highly unlikely that any of the optical measurements are related to this phenomenon.

<u>Analysis of variance: one-way classification</u>

Due to the qualitative nature of the stimuli it is difficult to compare quantitatively the responses of one subject with those of another. Conceding the presence of these difficulties due to individual differences, we can average the responses of the nine subjects for each of the eight types of conditions referred to on pages 35 and 36 and determine whether there are significant differences among the sets.

The real problem is to determine whether the sets of data obtained for each pair of colors under the various experimental conditions (lateral relation of target pairs and alignment toward and away) are sufficiently homogeneous to be regarded as belonging to the same population.

In other words we must first test the raw data in order to see whether or not position of variable, to right or left, from near to far or far to near, introduced systematic differences over and above those sought for in the experimental design. Whether or not we can combine findings of separate groups of experimental conditions into a larger composite distribution (for a two-way analysis of variance) hinges on the answer to this question. The terminology and analysis of this data follows the procedures described by Guilford (5, 236-259).

The total variance in a composite sample can be calculated as a one-way classification problem. The format for each of the three stimuli-combinations is shown in Table XII. The *X* and *Y* symbols represent any pair of targets and the four columns represent the spatial direction of movement made by the subject of the target which is to be aligned with the control. Incidentally, this method of analysis will yield a mean figure in each of the three conditions, namely red-green, red-blue and blue-green, indicating the amount by which a given color is perceived to deviate from the other member of a pair.

TABLE XII

FORMAT FOR ANALYSIS OF VARIANCE
ONE-WAY CLASSIFICATION PROBLEM

Position	Alignment			
	F forward	X backward	Y forward	Y backward
X-Y				
Y-X				

One example of this analysis of variance of position versus movement using the red-green combination can be found in Appendix II. The value of the obtained *F* is only 1.09, whereas the 5% level of confidence (for significance of differences) is 6.59. On the basis of these results we conclude in this case that there is no significant difference whether the combination is presented red-green or green-red or whether the movement is toward the subject or away from him.

The results of the three analyses are shown in Table XIII. They indicate that a specific lateral position (left or right) of any target has no bearing on the results, and it makes no difference whether the alignment is made from a more proximal or more distal position. From this data we can conclude that there are no significant differences among the sets of data that could be ascribed to the method of presentation of the stimuli. Since no difference exists we can combine the total distribution into a larger composite distribution for a two-way analysis of variance.

TABLE XIII

TOTAL VARIANCE DIVIDED INTO TWO COMPONENTS

Components	Between sets	Within sets	F	Confidence levels	
Degrees of freedom	3	4	7	5%	1%
Red-green	806.00	738.00	1.09	6.59	16.69
Red-blue	517.24	1247.50	.41	6.59	16.69
Blue-green	321.00	57.00	5.63	6.59	16.69

<u>Analysis of variance: two-way classification</u>

In order to appraise some of the inconsistent individual responses a two-way classification of analysis of variance was employed. The result is expected to show whether and to what degree some of the variance could be ascribed to persons, to the stimulating conditions, or to interaction between them. The analysis of variance is set up in Appendix III.

The results of the analysis of variance (Table XIV) indicates that differences in persons bear a highly significant relation to test scores. The F ratio is 29.19, well above the 1% level of confidence. Variations in results are influenced by the stimulus conditions (row two) and are significantly associated with variations in these conditions at the 1% level of confidence, since the F ratio is 30.26. The difference in results from one method to another would be expected to arise by random sampling fewer than once in a hundred times.

Much of the variance in the obtained scores is dependent on the interaction of a particular person with a particular pair of stimuli. Since this result is well above the 1% level of confidence (the F ratio is 42.41), it suggests that there are subjective differences in reaction to color.

TABLE XIV

RESULTS OF ANALYSIS OF VARIANCE

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F ratio	Required F ratio 5%	1%
Between rows (persons)	5,654	8	706.75	29.19	1.98	2.60
Between columns (stimulus pairs)	1,465	2	732.50	30.26	3.04	4.71
Interaction	16,428	16	1026.75	42.41	1.69	2.09
Within sets	4,576	189	24.21			
Total	28,123	215				

Composite spatial relations

The derived means for the three combinations of targets are each based on a total of seventy-two readings (eight adjustments for nine subjects). Table XV indicates that on the average the blue color is seen 64.5 mm. closer than the green and 3.2 mm. closer than the red, while the red is seen 16.5 mm. closer than the green. This is a consistent relationship.

A "t" was calculated for each of the three differences; the blue-green difference was found to be highly significant. The red-green difference was significant at the 5% level, and there was no statistically significant difference between the blue and the red.

TABLE XV

COMPOSITE SPATIAL RELATIONS

Pair	Closer	Farther	Distance (mm.)	S.E. of Mean	Level of Significance t	%
Blue-green	Blue	Green	64.5	4.60	9.910	1%
Blue-red	Blue	Red	3.2	9.66	2.299	None
Red-green	Red	Green	16.5	9.69	2.810	5%

Levels of confidence: 1% = 3.499; 5% = 2.365

These results are similar to those obtained in the phenomenal response from the fifty-two subjects. It may be recalled (pages 37 and 38) that red and blue were significantly selected as the proximal colors over the green; and the results of the chi-square test (Table IX, page 39) showed no significant difference in the blue and red distributions.

These results cannot be attributed to chance alone, therefore another factor must be sought which will help explain the data further or expose some other causal feature.

<u>Removal of sources of variation</u>

To appraise the different kinds of variance and the way in which they contribute to the total variance in the sample they can be separated in another way. Table XVI shows the twenty-seven means of the sets for the data. Variations among them are due to three possible sources, individual differences, target differences and the interaction of the two.

The possible effects of the individual differences are most apparent in the means of the rows, 45, 39, 30, etc. The possible effects of target differences are most apparent in the means of the columns, 33, 31 and 37. The possible interaction variance is obscured. It possibly contributes to both the means of the rows and of columns, we do not know. If we strip away the variations attributable to persons and then that attributable to targets, we will have the variation due to interaction.

TABLE XVI

REMOVAL OF SOURCES OF VARIATION

A. Original Matrix of Means of Sets

Subject	Red-green	Red-blue	Blue-green	Mean
BR	31	52	52	45
DA	24	50	43	39
AR	30	30	31	30
PE	27	40	37	35
KO	41	7	41	30
BE	44	13	31	29
LU	37	23	30	30
TR	27	39	38	35
TA	31	28	32	30
Mean	33	31	37	34

B. With Variations Removed; Interaction Variance Remaining

Subject	Red-green	Red-blue	Blue-green	Mean
BR	23	46	40	34
DA	20	48	35	34
AR	34	36	31	34
PE	27	42	33	34
KO	46	14	42	34
BE	49	20	32	34
LU	42	30	31	34
TR	27	41	34	34
TA	35	34	32	34
Mean	34	34	34	34

In Table XVI B most cells show a departure from the mean of thirty-four. These departures represent the interaction variation depending on whether or not they prove significant. The red-blue stimulus (column two) seems to have high scores with subjects BR, DA, PE and TR and to favor low scores with subjects KO, BE and LU. The other stimulus combinations show similar results differing only in degree. These results agree with the results of the F test which showed these deviations to be highly significant.

The two most consistent subjects in this Table XVI B, showing the least interaction, appear to be AR and TA. It may be recalled that both these subjects had difficulty in identifying the spatial order initially, and both gave contradictory data in the subjective-objective relations.

CHAPTER V

DISCUSSION AND CONCLUSIONS

An attempt should be made to evaluate the results of this experiment in accordance with the levels of optics, physiology and psychology. Following that, conclusions can be drawn from the data irrespective of the hypothesis advanced on the level advanced. The results will then be compared to other related studies in the literature.

The premises advanced by Einthoven and Hartridge to explain chromatic-stereo-effects are essentially physical in nature, being related to aberrations within the eye as a consequence of angle alpha. The aim of the present study was to test the hypothesis that angle alpha was the causal agent for the stereo-effect. If was expected that the greater the angle, the greater the effect, and that a positive significant correlation would be found. Since visual discrimination was involved the refractive error and the pupillary distance were noted.

The correlation results on page 43 show that the various classes of responses bear no relationship to the pupillary distance, refractive error or angle alpha. We can conclude that the stereo-effect is independent of refractive status; myopes, hyperopes and emmetropes in each group fail to give concordant or consistent results.

The varying values of angle alpha show no relation to quality (let alone quantity) of response; persons with a high positive angle may see either the red or the blue as closer and may even change from day to day. Thus, the hypothesis advanced in this experiment was disproved and it would appear that none of the physical factors considered admit of a rational explanation of the findings.

Other experimental studies have shown similar results. Among his nine subjects, Luckiesh (10) found two who did not conform to the results obtained from the rest. Recently, Karwoski and Lloyd (9), in studying the role of chromatic aberration in depth perception found that their subjects were most accurate with yellow in identifying that color against a criterion in a darkened room, and that they were poorest with blue. They submit averages for ten observers. Three of their ten subjects showed complete reversals which the authors attempt to discard on the basis of astigmatism and differences in acuity.

On the functional side attempts have been made to explain an accommodative response on the basis of chromatic aberration; this is not a valid consideration in this study since the distance was twenty feet and all the subjects were fully corrected making accommodation unnecessary.

Efforts to explain paradoxical reports on this basis of accommodation by Luckiesh (10), by Prentice, Krimsky and Barker (12), Karwoski (9) and Taylor and Sumner (14) are totally inadequate in view of the work of Sheard (13) who showed that light and color (per se) from spectral filters supply very little stimulus to accommodation.

In pursuing the physiological aspect further one cannot say that the underlying physiological mechanism is the same in all persons; and if it varies from one individual to another, or fluctuates in the same individual, this would result in the individual differences found, but we have no concrete evidence for this.

The preliminary experiment demonstrated that brightness is a complicating factor in distance perception of chromatic objects. The data from the phenomenal experiment, substantiated by the controlled experiment leads to the conclusion that perception of distances based on chromatic effects varies markedly in different subjects, and varies from time to time in the same subject. Some subjects show consistency in their responses but others are quite inconsistent and contradictory. The suggestion is advanced that stereo-judgments may be a function of the stimulating situation and the functional state of the individual, so the problem should be approached from the standpoint of psychological differences and representative design of the experiment.

Although the hypothesis was disproved further analysis of the data seemed indicated. The one-way analysis of variance showed that the results were not affected by the method of presentation of the stimuli. The composite spatial relations averaged for the subjects showed a significant factor operating on a statistical level, particularly where the green was involved with the red and the blue; with no difference between the red-blue matches. The two-way analysis of variance yielded significant information, and showed that interaction of persons with chromatic stimuli contributed the greatest variation. On this basis the conclusion is justified that the individual differences and responses are highly subjective and that chromatic stereo-effects appear to be a purely psychological phenomenon. On stripping away the sources of variation from targets and from persons a great deal of interaction variance was demonstrated to exist (Table XVI, page 50).

If reality consists of experiences of perceptual phenomena according to Gestalt, more emphasis should be placed on the subjective aspects of this experience. This raises the question of whether experience is the same for all persons.

This experiment, and others cited, indicate that there are subjective differences in reaction to color which should be investigated further. If people respond to color effects differently their color perception should differ as well.

Suggested investigations

The results of this study have prompted several new aspects of the problem which might be investigated profitably. Two suggestions are;

(1) that this study be repeated under the same conditions using squares of "white" light of the same brightness as that of the chromatic squares. If color is a real factor a significant difference should be obtained between the two sets of data; if not, no statistically significant difference would be obtained.

(2) that the same study be conducted on two groups of subjects, (a) a control group of persons who are known to be red-green color blind (protanopes and deuteranopes), and (b) an experimental group consisting of persons who have normal color vision.

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APPENDIX I

SUBJECTIVE RESPONSES TO COLORED DISCS VIEWED AGAINST DIFFERENT BACKGROUNDS

N = 77

Black background		White background	
Response	Number	Response	Number
Y-B	29	B-Y	23
Y-P	7	B-G	8
Y-R	6	R-Y	18
R-B	7	R-B	4
R-G	2	R-G	1
G-B	5	G-Y	5
G-P	5	P-Y	4
O-B	2	P-B	1
P-B	1	O-B	2
P-G	2	Y-B	2
P-O	1		
Total	67	Total	68
Alternations	10	Alternations	7
		No effect	2
N	77	N	77

APPENDIX I (continued)

SUBJECTIVE RESPONSES TO COLORED DISCS
VIEWED AGAINST DIFFERENT BACKGROUNDS

Code:	R:	red	G:	green
	O:	orange	B:	blue
	Y:	yellow	P:	purple

Only the nearest and farthest colors were considered in this tally; the nearest color being listed first in the above.

APPENDIX II

ANALYSIS OF VARIANCE OF POSITION VS. MOVEMENT
RED[redacted] DATA USED FOR EXAMPLE

	RF	RB	GF	GB	
R-G	0	2	42	40	
G-R	36	52	84	52	
ΣX_s	36	54	126	92	ΣX = 308.
M_s	18	27	63	46	M_t = 38.50*
Deviations Within Sets (x_s)					
	-18	-25	-21	-6	
	18	25	21	6	
Squares of the deviations					
	324	625	441	36	
	324	625	441	36	
	648	1350	882	72	Σx^2_s = 2952 (Sum of within variance)
Deviations of set means from grand mean (d)					
d	-20.50	-11.50	24.50	7.50	
d^2	420.25	132.25	600.25	56.25	Σd^2 = 1209.00
nd^2	840.50	264.50	1200.50	112.50	$n\Sigma d^2$ = 2418.00 (Sum of between variance)

* This mean is derived from coded data. Twenty-two must be subtracted, so the obtained mean is 16.50.

APPENDIX II (continued)

ANALYSIS OF VARIANCE OF POSITION VS. MOVEMENT
RED-GREEN DATA USED FOR EXAMPLE

Total Variance			
Components	df	Sum of squares	Variance
Between sets (k) Within sets	3 4	2418.0 2952.0	806.00 738.00
$F = \dfrac{806.00}{738.00} = 1.09$			Table: 1% = 16.69

No significant difference exists whether position is R-G or G-R, or whether target is moved in or out.

APPENDIX III

TWO-WAY ANALYSIS OF VARIANCE

	Red-green		Red-blue		Blue-green			
Sub-ject	X	X^2	X	X^2	X	X^2	ΣX_r	$\Sigma(X^2_r)$
BR	28	784	62	3844	51	2601		
	28	784	57	3249	53	2809		
	35	1225	43	1849	57	3249		
	34	1156	58	3364	48	2304		
	28	784	58	3364	48	2304		
	35	1225	46	2116	51	2601		
	31	961	49	2401	55	3025		
	30	900	43	1849	55	3025		
Σ	249	7819	416	22036	418	21918	1083	51773
DA	22	484	55	3025	47	2209		
	22	484	53	2809	46	2116		
	31	961	50	2500	48	2304		
	29	841	51	2601	43	1849		
	19	361	47	2209	43	1849		
	25	625	51	2601	43	1849		
	24	576	46	2116	40	1600		
	21	441	47	2209	45	2025		
	193	4773	400	20070	355	15801	948	40644
AR	26	676	34	1156	29	841		
	26	676	34	1156	30	900		
	28	784	30	900	32	1024		
	27	729	32	1024	28	784		
	27	729	28	784	30	900		
	33	1089	28	784	32	1024		
	38	1444	26	676	33	1089		
	34	1156	26	676	34	1156		
	239	7283	238	7156	248	7283	725	21722

APPENDIX III (continued)

TWO-WAY ANALYSIS OF VARIANCE

	Red-green		Red-blue		Blue-green			
Subject	X	X^2	X	X^2	X	X^2	ΣX_r	$\Sigma(X^2_r)$
PE	20	400	46	2116	32	1024		
	22	484	42	1764	33	1089		
	22	484	46	2116	35	1225		
	25	625	46	2116	32	1024		
	33	1089	33	1089	42	1764		
	34	1156	36	1296	44	1936		
	33	1089	34	1156	38	1444		
	30	900	37	1369	39	1521		
	219	6227	320	13022	295	11027	834	30276
KO	41	1681	9	81	48	2304		
	40	1600	6	36	49	2401		
	53	2809	13	169	43	1849		
	51	2601	4	16	40	1600		
	33	1089	8	64	32	1024		
	33	1089	9	81	29	841		
	48	2304	0	0	44	1936		
	33	1089	6	36	46	2116		
	332	14262	55	483	331	14071	718	28816
BE	40	1600	15	225	32	1024		
	39	1521	11	121	33	1089		
	38	1444	23	529	36	1296		
	38	1444	21	441	28	784		
	60	3600	8	64	28	784		
	45	2025	8	64	36	1296		
	48	2304	11	121	26	676		
	47	2209	9	81	26	676		
	355	16147	106	1646	245	7625	706	25418

APPENDIX III (continued)

TWO-WAY ANALYSIS OF VARIANCE

	Red-green		Red-blue		Blue-green			
Subject	X	X^2	X	X^2	X	X^2	ΣX_r	$\Sigma(X^2_r)$
LU	35	1225	27	729	23	529		
	32	1024	23	529	28	784		
	30	900	25	625	28	784		
	30	900	25	625	25	625		
	36	1296	18	324	32	1024		
	42	1764	23	529	34	1156		
	45	2025	22	484	37	1369		
	47	2209	22	484	32	1024		
	297	11343	185	4329	239	7295	721	22967
TR	21	441	43	1849	32	1024		
	25	625	47	2209	34	1156		
	28	784	42	1764	38	1444		
	25	625	26	676	41	1681		
	28	784	44	1936	38	1444		
	29	841	36	1296	39	1521		
	32	1024	39	1521	44	1936		
	27	729	36	1296	40	1600		
	215	5853	313	12547	306	11806	834	30206
TA	27	729	30	900	35	1225		
	26	676	36	1296	34	1156		
	32	1024	17	289	41	1681		
	37	1369	24	576	38	1444		
	29	841	31	961	28	784		
	28	784	29	841	28	784		
	36	1296	27	729	30	900		
	37	1369	28	784	21	441		
	252	8088	222	6276	255	8415	729	22879

APPENDIX III (continued)

TWO-WAY ANALYSIS OF VARIANCE

	Red-green		Red-blue		Blue-green			
Subject	X	X^2	X	X^2	X	X^2	ΣX_r	$\Sigma(X^2{}_r)$
ΣX_k	2351		2255		2692		$7298 = \Sigma X_{ij}$	
$\Sigma X^2{}_k$		81795		87665		105,241	$\Sigma X^2{}_{ij} = 274{,}701$	

Note: All the X scores in the above table have been coded. To obtain the original raw score, subtract 31 and multiply by 10. This will give the reading in millimeters.

APPROVAL SHEET

The thesis submitted by Thaddeus Rudolph Murroughs has been read and approved by three members of the Department of Psychology.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Arts.

May 28, 1954
Date

V.V. Herr, S.J.
Signature of Adviser